

# Determining Quality Inspection Frequency in an Automated Production Line Based on Field Failure Data Analysis

Panagiotis Tsarouhas

Department of Informatics and Computer Technology, Technological Education Institute of Lamia, Greece, e-mail: [ptsarouh@teilam.gr](mailto:ptsarouh@teilam.gr)

George Liberopoulos

Department of Mechanical & Industrial Engineering, University of Thessaly, Greece, e-mail: [glib@mie.uth.gr](mailto:glib@mie.uth.gr)

We study the problem of determining the frequency of quality control inspections in a tortilla and bread & bakery manufacturer producing pizzas. We first perform statistical analysis of failure data obtained from a real automated pizza production line. Based on this data, we develop a simple model of a quality inspector who visits several such lines and his goal is to allocate the number of his/her visits to the different workstations of the lines so as minimize the total production time of undetected, defective products.

**Keywords** Production; Reliability; Quality; Optimization

## 1 Introduction

Failures that lead to product quality deterioration in automated high-volume production lines are an important managerial concern in manufacturing. Such failures may be due to human factors such as fatigue, distractions, low skill levels, job dissatisfaction, high turnover and absenteeism, and undesirable job attitudes and behaviors. Non-human factors, such as poorly maintained machine/equipment, defects in raw materials, and so on, can also lead to failures that affect product quality. The cost of these failures can be categorized into internal and external failure costs. Internal failure costs are associated with correcting a defect before the customer receives the item. These costs include costs related to scrap, rework, lost labor hours and machine capacity, failure analysis, re-inspection and retesting, downgrading, and increased lead times and inventory. External failure costs are associated with defects that are discovered after the product is shipped to the customer. These costs include warranty repairs and replacements and the loss of customer good will. They may also include costs related to legal liability and lawsuits.

Product quality is usually appraised and controlled by quality inspectors at all the workstations of a production line during the production process. The duty of a quality inspector is to detect failures that cause defects and subsequently worsen product quality. Once such a failure is detected, the inspector stops the line and calls maintenance technicians and workers to repair the failure. Many manufacturing companies can not afford to have a quality inspector at each workstation of a production line. A typical situation in the food industry, for example, is that of one quality inspector who is in charge of several lines, which he/she visits during his/her shift. If a failure that affects product quality takes place at one of the workstations of a line, the inspector will not detect it until his/her next visit to the point of failure. During the interval from the time of failure to the time of inspection and detection, defective products are being produced.

Although the literature on quality control and on maintenance is substantial, the interconnection between quality and maintenance has been somewhat overlooked. An integrated cost model for the joint optimization of process control and maintenance was developed in [Tagaras (1988)]. The relation and interaction between maintenance, production, and quality was presented in [Ben-Daya and Duffuaa (1995)]. In [Ahire et al. (1995)], it was observed that a vast majority of literature on quality issues overlooked the relationship between quality improvement strategies and their associated financial performance measures. In [van der Bij and van Ekert (1999)] it was shown that a systematic design of quality control systems is necessary for the quality performance as well as the performance on the production control. An integrated model for the joint optimization of production quality, design of quality control parameters, and maintenance level was studied in [Ben-Daya (1999)]. Another approach was that taken in [Burney and Al-Darrab (1998)], who considered an application of statistical quality control techniques on human performance evaluation in a service industry. Their study included data description, collection and analysis, and the development of a subjective performance evaluation criterion. The effectiveness of the line-stop on-line repair policy over traditional off-line repair policies for an assembly line was examined in [Shin and Min (2001)] through two mathematical models based on total quality failure costs.

In this paper we study the problem of determining the frequency of quality control inspections in a tortilla and bread & bakery manufacturer producing pizzas. We first perform statistical analysis of failure data obtained from a real automated pizza production line. Based on this data, we then develop a simple model of a quality inspector who visits several such lines and his goal is to allocate the number of his/her visits to the different workstation of the lines so as minimize the total production time of undetected defective products.

The production lines that make bread & bakery products are similar for a wide range of different product types, such as breads, bagels, doughnuts, pastries, bread-type biscuits, toasts, cakes, crullers, croissants, pizzas, knishes, pies, rolls, buns, etc. (e.g., see [Liberopoulos and Tsarouhas (2002) for a description of a croissant production line). Consequently, the analysis in this paper, although it focuses on pizzas, applies to most types of bread & bakery products.

## **2 Description of an Automated Pizza Production Line**

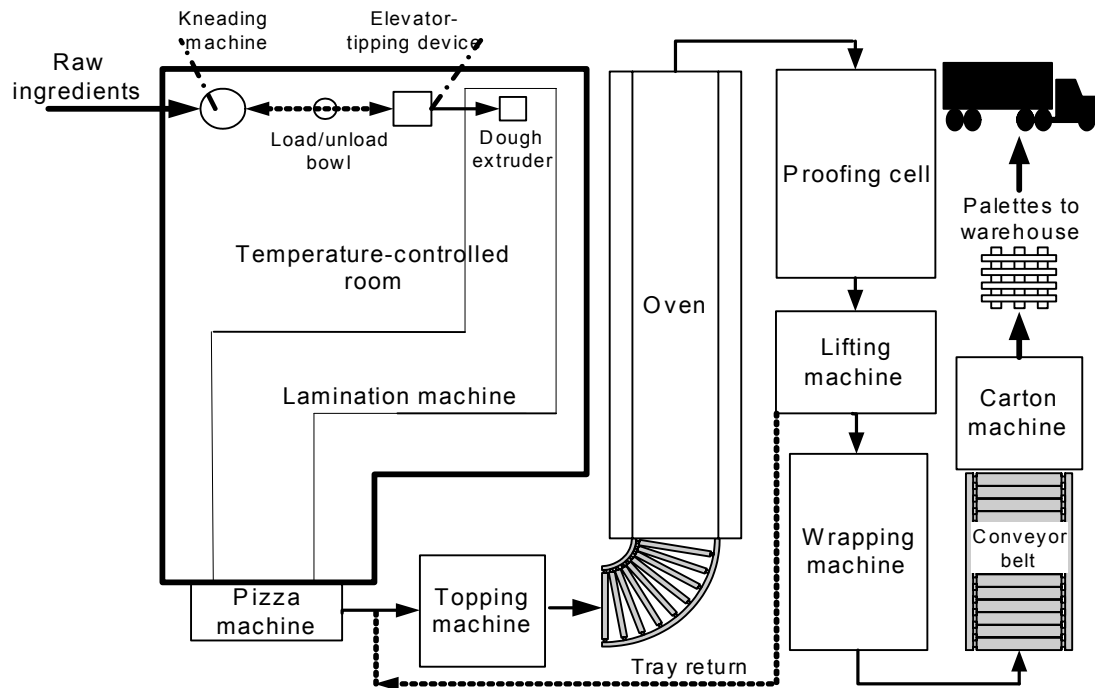
An automated pizza production line consists of several workstations in series integrated into one system by a common transfer mechanism and a common control system. The movement of material between stations is performed automatically by mechanical means. A schematic representation of a pizza production line is shown in Figure 1. There are six distinct stages in making pizzas: kneading, forming, topping, baking, proofing, and wrapping. Each stage corresponds to a distinct workstation, as follows.

In workstation 1, flour from the silo and water are automatically fed into the removable bowl of the spiral kneading machine. Small quantities of additional ingredients such as sugar and yeast are added manually. After the dough is kneaded, the bowl is manually unloaded from the spiral machine and loaded onto the elevator-tipping device that lifts it and tips it to dump the dough into the dough extruder of the lamination machine in the next workstation.

In workstation 2, the dough fed into the lamination machine is laminated, folded, reduced in thickness by several multi-roller gauging stations to form a sheet. The sheet is then automatically fed into the pizza machine, which cuts it into any shape (usually a circle or a square) with a rotary cutting roller blade or guillotine. The entire process is fully automated. At the exit of the pizza machine, the pizzas are laid onto metal baking pans that are automatically fed to the next workstation.

In workstation 3, tomato sauce, grated cheese and other toppings, such as vegetables, ham, pepperoni cubes and sausage, are automatically placed on the pizza base by a target

topping system leaving a rim free of topping. One of the reasons that the toppings are placed on the pizza base before the pizza is baked is to prevent the pizza base from rising.



**Figure 1** Schematic Representation of a Pizza Processing Line

In workstation 4, the baking pans are placed onto a metal conveyor which passes through the baking oven. The pans remain in the oven for a precise amount of time until the pizzas are partially or fully baked. Extra toppings are optionally placed on top of the pizzas at the exit of oven (usually for partially backed pizzas).

In workstation 5, the baking pans are collated together and fed into the proofer entrance. As soon as they enter the proofer, they are moved onto the stabilized proofer trays by means of a pusher bar. The proofer trays are automatically transported inside the proofer by conveyors and paternoster-type lifts in order for the pizzas to cool down and stabilize. The baking pans are pushed off the stabilized proofer trays onto the outfeed belt and are automatically transported out of the proofer.

In workstation 6, the pizzas are automatically lifted from the baking pans and are flow-packed and sealed by a horizontal, electronic wrapping machine. The empty pans are automatically returned to the pizza machine. The final products that exit from the pizza production line are loaded onto a conveyor. From there, they are hand-picked and put in cartons. The filled cartons are placed on a different conveyor that takes them to a worker who stacks them on pallets and transfers them to the finished-goods warehouse.

### 3 Descriptive Statistics of Field Failure Data

Production managers routinely record failure data for the systems they manage as they use these systems for their intended purposes and maintain them upon failure. We had access to such data from a pizza production line of a large tortilla and bread & bakery manufacturer. The line is identical to that described in the previous section. It consists of six workstations in series, where each workstation contains one or more machines, and each machine has several failure modes.

To take into account exogenous failures affecting the entire line, we define a seventh pseudo-workstation and call it “exogenous.” The exogenous workstation has four pseudo-machines, which correspond to the electric, water, gas, and air supply, respectively. Each

pseudo-machine has a single failure mode corresponding to a failure in the supply of one of the four resources mentioned above. Failures at workstation 7 are very important because they affect the entire line. The most significant of these failures is the failure of the electric power generator that temporarily supplies the system with electricity in case of an electric power outage. Throughout the paper we use the notation:

$F.i.j.k$  = Failure mode  $k$  of machine  $j$  of workstation  $i$ .

The failure data that we had access to covers a time period of 1491 days, i.e. four years and one month. During this period, the line operated for 24 hours a day, with three eight-hour shifts during each day, for a total of 883 working days. The data was extracted from the hand written records of failures that the maintenance personnel kept during each shift. The records included the failure mode or modes that had occurred during the shift, the action taken, the down (repair) time, but not the exact time of failure. This means that our accuracy in computing the *time between failures* (TBF) of a particular failure mode, machine, workstation, or of the entire line itself is in the order of number of eight-hour shifts rather than in the order of number of hours. The *time to repair* (TTR), on the other hand, was recorded in minutes. From the records, we counted a total of 1772 failures for the entire line, which were classified into 203 different failure modes that interrupted production. Besides these failure modes, there were 13 additional failure modes, which had no direct effect on production and were thus excluded from the data. From the records, we computed several important descriptive statistics of the failure data at the levels of the failure modes, the machines, the workstations, and finally the entire line. The sample size for computing the parameters of TBF is one less than the number of failures, whereas the sample size for computing the parameters of the TTR is equal to the number of failures.

The descriptive statistics of the failure data and the resulting availability at the workstation and production line levels are presented in [Liberopoulos and Tsarouhas (2003)], where the availability is computed as the ratio of the mean TBF over the sum of the mean TBF plus the mean TTR. The analysis in [Liberopoulos and Tsarouhas (2003)] shows that the three workstations with the most frequent failures and lowest availabilities are workstations 2, 5, and 6, in decreasing order of failure frequency and increasing order of availability. Indeed, the most failure-prone workstation, 2, is at the heart of the production process and consists of a very complex set of equipment with a total of 52 different failure modes.

Not all failures have a direct effect on product quality. Out of the 203 different failure modes, there are 75 failure modes that directly affect product quality. These failure modes occurred 777 times. Table 1 lists the 25 most important failure modes affecting product quality, where by important we mean that they occurred more than eight times. The descriptive statistics of these failure modes are shown in Table 2.

**Table 1** Description of the Most Important Failure Modes Affecting Product Quality

<b>Failure mode</b>	<b>Description</b>	<b>Effect on product quality</b>
F.2.1.2	Failure of flour machine	Unacceptable dough
F.2.1.8	Failure of cutting machine of dough	Uneven dough
F.2.1.11	Failure of butter pump	Unacceptable dough
F.2.1.13	Motion-chain at the lamination machine is broken	Unacceptable dough
F.2.1.14	Malfunction of butter pipe	Unacceptable dough
F.2.1.16	Failure of sensor at lamination machine	Unacceptable dough
F.2.1.18	Failure of the pneumatic system at butter machine	Unacceptable dough
F.2.2.8	Motion-chain at the pizza machine is out-of-phase	Unacceptable dough
F.2.2.10	Malfunction of the conveyor belt	Unacceptable shape of product
F.2.2.12	Failure of the rotary cutting roller or guillotine	Unacceptable shape of product
F.2.2.13	Realignment of laminated dough on the conveyor at the pizza machine	Unacceptable shape of product
F.2.2.15	Blocking of mechanism that lays pizzas onto metal	Unacceptable shape of product

	baking pans	
F.3.1.2	Failure of pneumatic system with pistons at the topping machine	Uneven topping and weight of product
F.3.1.9	Leaking gasket at the topping machine	Uneven topping and weight of product
F.3.1.23	Cleaning of malfunctioning nozzles of the topping machine	Uneven topping and weight of product
F.3.1.25	Cleaning of clogged nozzles at the topping machine	Uneven topping and weight of product
F.4.1.2	Broken motion-chain of metal conveyor	Burnt product
F.4.1.3	Blocking of pans in the oven	Burnt product
F.4.1.6	Failure of burner at the baking oven	Burnt product
F.5.3.1	Clogged nozzles at the cooling unit of the proofing section	Wrong freezing conditions
F.5.3.2	Cleaning of air filters at the transporter in the proofing section	Wrong freezing conditions
F.6.1.5	Failure of the forks that automatically lift pizzas from the baking pans	Unacceptable shape of product
F.6.1.16	Failure of the pneumatic system at lifting machine	Unacceptable shape of product
F.6.2.6	Malfunction of photocell at wrapping machine	Unacceptable wrapping of product
F.6.3.4	Adjustment of carton sealing mechanism	Unacceptable wrapping of product

**Table 2** Descriptive Statistics of the Most Important Failure Modes Affecting Product Quality

	N	Min	Max	Mean	Std. Dev.	Skewness	Std. Err.	Kurtosis	Std. Err.
TBF F.2.1.2	12	8	601	180.9167	171.5022	1.3583	0.6373	2.2014	1.2322
TBF F.2.1.8	13	3	789	183.7692	221.1339	1.8833	0.6163	4.1774	1.1909
TBF F.2.1.11	13	2	698	152.2308	211.5452	1.6908	0.6163	2.6542	1.1909
TBF F.2.1.13	9	13	786	166.5556	238.1161	2.7345	0.7171	7.7791	1.3997
TBF F.2.1.14	13	11	1083	183.4615	289.1348	2.8904	0.6163	9.0810	1.1909
TBF F.2.1.16	25	0	506	101.92	137.0812	2.0767	0.4637	4.0875	0.9017
TBF F.2.1.18	9	2	872	260	325.8006	1.3831	0.7171	0.4326	1.3997
TBF F.2.2.8	74	0	531	34.52703	75.16762	5.0789	0.2792	29.0601	0.5517
TBF F.2.2.10	17	3	856	143.4706	254.1302	2.3713	0.5497	4.7216	1.0632
TBF F.2.2.12	30	1	798	78.53333	152.1784	3.9972	0.4269	18.0802	0.8327
TBF F.2.2.13	73	1	339	34.68493	56.67374	3.6570	0.2810	15.8229	0.5552
TBF F.2.2.15	23	2	697	104.8696	146.3332	3.2382	0.4813	12.6987	0.9348
TBF F.3.1.2	39	1	151	31.38462	31.77355	1.8748	0.3782	4.2639	0.7410
TBF F.3.1.9	19	3	754	110.8947	176.8794	3.0341	0.5238	10.2723	1.0143
TBF F.3.1.23	15	7	637	149.0667	194.1261	1.4581	0.5801	1.3264	1.1209
TBF F.3.1.25	19	1	690	128.8421	192.3236	2.2057	0.5238	4.4051	1.0143
TBF F.4.1.2	12	3	1457	181.8333	408.2526	3.2724	0.6373	10.9948	1.2322
TBF F.4.1.3	16	3	739	132	181.2196	2.7954	0.5643	8.9891	1.0908
TBF F.4.1.6	55	1	415	46.81818	77.44654	3.6167	0.3217	14.4374	0.6335
TBF F.5.3.1	43	0	424	53.25581	80.6722	2.9255	0.3614	10.3639	0.7090
TBF F.5.3.2	9	1	1540	245.2222	501.9198	2.6767	0.7171	7.3586	1.3997
TBF F.6.1.5	27	0	420	88.59259	116.0926	1.9073	0.4479	2.9752	0.8721
TBF F.6.1.16	9	7	887	245.7778	336.2182	1.4023	0.7171	0.4505	1.3997
TBF F.6.2.6	10	2	599	230.8	182.7426	0.6967	0.6870	0.3418	1.3342
TBF F.6.3.4	51	1	680	49.56863	111.6696	4.3632	0.3335	21.4851	0.6559
TTR F.2.1.2	13	20	40	26.92308	5.964639	0.4964	0.6163	0.3208	1.1909
TTR F.2.1.8	14	30	50	41.78571	4.643914	-0.8572	0.5974	2.5930	1.1541
TTR F.2.1.11	14	20	35	28.92857	3.496466	-1.2533	0.5974	2.8763	1.1541
TTR F.2.1.13	10	25	45	35	7.071068	0.2946	0.6870	-1.3929	1.3342
TTR F.2.1.14	14	25	35	31.42857	3.056249	-0.1925	0.5974	-0.2576	1.1541

TTR F.2.1.16	26	45	120	60.57692	17.51153	2.0868	0.4556	4.9122	0.8865
TTR F.2.1.18	10	30	40	34	3.944053	0.4075	0.6870	-1.0742	1.3342
TTR F.2.2.8	75	40	90	61.13333	9.849315	0.3830	0.2774	0.1905	0.5482
TTR F.2.2.10	18	30	60	35.27778	7.759666	2.0214	0.5363	5.2725	1.0378
TTR F.2.2.12	31	30	40	33.87097	4.417706	0.4764	0.4205	-1.5821	0.8208
TTR F.2.2.13	74	10	30	17.7027	4.070373	1.3534	0.2792	1.7061	0.5517
TTR F.2.2.15	24	50	80	59.58333	7.928576	0.7194	0.4723	0.4564	0.9178
TTR F.3.1.2	40	10	30	17.75	3.571612	0.4821	0.3738	2.4867	0.7326
TTR F.3.1.9	20	15	40	21.25	7.758696	1.6023	0.5121	1.8357	0.9924
TTR F.3.1.23	16	10	60	19.375	14.47699	1.9875	0.5643	3.4005	1.0908
TTR F.3.1.25	20	10	40	19.75	9.101041	1.3037	0.5121	0.9157	0.9924
TTR F.4.1.2	13	15	30	19.61538	5.188745	1.2327	0.6163	0.9286	1.1909
TTR F.4.1.3	17	15	60	30.29412	12.8051	1.0900	0.5497	0.4457	1.0632
TTR F.4.1.6	56	15	60	23.30357	9.208533	1.8082	0.3190	4.1904	0.6283
TTR F.5.3.1	44	20	40	24.77273	5.999471	1.1063	0.3575	0.5391	0.7017
TTR F.5.3.2	10	20	40	26	6.992059	1.0848	0.6870	0.2648	1.3342
TTR F.6.1.5	28	10	30	16.25	5.713046	0.7520	0.4405	0.4234	0.8583
TTR F.6.1.16	10	20	50	32	12.29273	0.4307	0.6870	-1.4612	1.3342
TTR F.6.2.6	11	15	20	16.81818	2.522625	0.6607	0.6607	-1.9643	1.2794
TTR F.6.3.4	52	10	20	12.40385	2.885118	0.7143	0.3304	-0.4532	0.6501

From Tables 1 and 2 we can make the following observations. Most of the important failure modes affecting product quality belong to workstation 2. Three of these failure modes, F.2.1.16, F.2.2.8, and F.2.2.15, have the largest mean TTR. Some of the failure modes affecting product quality occur frequently but have fast repair times. These failures usually involve a minor adjustment or the cleaning of equipment (e.g., F.2.2.13, F.6.3.4, and F.5.3.1).

#### 4 Model Formulation and Analysis

As was mentioned above, the failure data that we had access to was extracted from the hand written records of failures that the maintenance personnel kept during each shift. The records included the failure mode or modes that had occurred during the shift, the action taken, the down (repair) time, but not the exact time of failure. Based on the failure data, we develop a simple model of a quality inspector who visits several pizza production lines and his/her goal is to allocate the number of his/her visits to the different workstation of the lines so as minimize the total production time of undetected defective products. All the lines are identical, so we restrict our attention to any one line. To introduce the mathematical model, we use the following notation:

- $M$  = Number of failures that directly affect product quality (integer);
- $N_i$  = Number of inspections per shift to detect failure mode  $i$  (integer);
- $N_{\max}$  = Maximum total number of inspections per shift (integer);
- $F_i$  = Mean TBF of failure mode  $i$ ;
- $T_i$  = Time to detect failure mode  $i$  in a shift when there are  $N_i$  inspections per shift;
- $X_i$  = Binary random variable taking the value 1 with probability  $P_i(t)$ , if failure mode  $i$  has occurred during a shift, and zero with probability  $1 - P_i(t)$ , if failure mode  $i$  has not occurred during a shift, where  $t$  is the number of shifts since the last failure;

The probability  $P_i(t)$  is equal to the probability that the time to failure is less than  $t + 1$  shifts, given that it is greater than  $t$  shifts and is given in terms of the reliability function  $R(t)$  as follows:

$$P_i(t) = [R(t) - R(t + 1)]/R(t). \quad (1)$$

We make the following assumptions:

1. The exact time of a recorded failure in a shift is uniformly distributed between 0 and 8 hours.
2. The TBF of failure mode  $i$  is exponentially distributed.

Assumption 1 states that a recorded failure may have taken place at any time during a shift with equal probability. This is a reasonable assumption, since, as was mentioned above, the maintenance personnel did not record the exact time of failures. Assumption 1 implies that  $E[T_i|X_i = 0] = 0$  and  $E[T_i|X_i = 1] = 4/N_i$ . Assumption 2 simplifies the exposure, because it implies that  $P_i(t)$  is independent of  $t$  and equal to  $P_i = 1 - \exp(-1/F_i)$ , which for  $F_i \gg 1$  (as is our case) can be approximated by  $P_i = 1/F_i$ . In reality, however,  $P_i(t)$  should depend on  $t$ . In fact, in [Liberopoulos and Tsarouhas (2003)], it is shown that the probability distribution which best fits the TBF data is the Weibull distribution. If the TBF of failure mode  $i$  has a Weibull distribution, with scale and shape parameters  $\theta_i$  and  $\beta_i$ , respectively, then  $P_i(t)$  is given by  $P_i(t) = [\exp(-(t/\theta_i)^{\beta_i}) - \exp(-((t+1)/\theta_i)^{\beta_i})] / \exp(-(t/\theta_i)^{\beta_i})$ . We can easily accommodate this case in our model at the cost of introducing time dependence.

Under Assumptions 1 and 2, the expected time to detect failure mode  $i$  in a shift when there are  $N_i$  inspections per shift can then be calculated as follows:

$$E[T_i] = E[T_i|X_i = 1] P[X_i = 1] + E[T_i|X_i = 0] P[X_i = 0] = 4/(N_i F_i).$$

We can now formulate a simple constrained optimization problem whose objective is to minimize the total expected time to detect a failure mode that affects product quality in a shift subject to the constraint that the total number of inspections per shift to detect such failures has an upper bound  $N_{\max}$ , i.e.

$$\min \sum_{i=1}^M \frac{4}{N_i F_i} \quad \text{subject to} \quad \sum_{i=1}^M N_i \leq N_{\max}. \quad (2)$$

To determine the value of  $N_{\max}$  we take into account the existing practice in the company. Currently, there is one quality inspector per shift. The inspector's duty is to check the product quality of 5 production lines and perform some laboratory tests. The inspector allocates approximately 6 hours of his/her 8-hour shift to on-line inspections and 2 hours in the laboratory. This means that the available time for inspection per shift per line is  $6 \cdot 60/5 = 72$  minutes per line per shift. The inspector spends approximately 4 minutes at each of the 6 workstations of a production line to check on all possible failure modes that may affect product quality. This means that he/she performs approximately  $72/4 = 18$  inspections per line per shift, which implies that on the average he/she visits each workstation of a line  $18/6 = 3$  visits per workstation per shift. This further implies that the total number of inspections for the 25 most important failure modes per shift is approximately  $25 \cdot 3 = 75$  inspections per line per shift. With this in mind, we set  $N_{\max} = 75$ . The optimization problem (2) then becomes:

$$\min \sum_{i=1}^{25} \frac{4}{N_i F_i} \quad \text{subject to} \quad \sum_{i=1}^{25} N_i \leq 75, \quad (3)$$

where the values of  $F_i$  are obtained from column 5 of Table 3.

The non-linear integer program (3) was solved using the optimization software LINGO. The result is that in the optimal solution, some failure modes still have to be inspected 3 times in a shift, as is the current practice, some other failure modes have to be

inspected 4 or even 5 times, whereas the remaining failure modes must be inspected only twice. Table 3 shows the 25 most important failures modes affecting product quality sorted according the optimal number of inspections per shift needed to detect them. The associated minimum total expected time to detect a failure mode that affects product quality in an 8-hour shift is 0.3379 hours. This means that  $0.3379/8 \approx 4.22\%$  of the products are defective. This constitutes an improvement over the current situation of having  $N_i = 3$  inspections per shift for each failure mode, where the total expected time to detect a failure mode that affects product quality in an 8-hour shift is 0.3762 hours, which implies that 4.70% of the products are defective. Finally, we should note that at the turn of each shift, the quality inspector performs a last check on each of the 7 workstations before handing over responsibility to the quality inspector of the next shift.

**Table 3** The Most Important Failures Modes Affecting Product Quality Sorted According to the Optimal Number of Inspections per Shift Needed to Detect Them

$N_i^* = 2$		$N_i^* = 3$		$N_i^* = 4$	$N_i^* = 5$
F2.1.2	F2.1.18	F2.1.16	F3.1.23	F.5.3.1	F.2.2.8
F2.1.8	F4.1.2	F2.2.10	F3.1.25	F.6.3.4	F.2.2.13
F2.1.11	F5.3.2	F2.2.12	F4.1.3		F.3.1.2
F2.1.13	F6.1.16	F2.2.15	F6.1.5		F.4.1.6
F2.1.14	F6.2.6	F3.1.9			

An important parameter of the model discussed above is the maximum total number of inspections per shift,  $N_{\max}$ . In our numerical example we used  $N_{\max} = 75$  based on the fact that currently each one of the 25 failure modes is inspected 3 times by the inspector. The number of inspections, even in the current situation, however, could be increased if the time allocated for inspections per shift were increased beyond 6 hours, or if the inspection time spent on each workstation were decreased below 4 minutes, or finally, if more inspectors were added. It is clear that if  $N_{\max}$  is multiplied by a factor of  $\alpha$  and the non-linear program (2) is resolved, the minimum total expected time to detect a failure mode that affects product quality in an 8-hour shift and therefore the percentage of defective products will be divided by the same factor  $\alpha$ . For example if  $N_{\max}$  is doubled from 75 to 150, the percentage of defective products will be reduced to  $\frac{1}{2}$  of its value from 4.22% to 2.11%.

## 5 Conclusions

We developed a simple model of a quality inspector who visits several such lines and his goal is to allocate the number of his/her visits to the different workstation of the lines so as minimize the total production time of undetected defective products. It was shown that if the number of inspections to detect different failure modes is not the same for each failure mode, as is the current situation, but is optimized, the percentage of defective products due to the delayed detection of a failure can be reduced from 4.70% to 4.22%. In the model that we developed we made several simplifying assumptions. One of these assumptions is that the time of a recorded failure in a shift is uniformly distributed between 0 and 8 hours. Another assumption is that  $P_i(t)$  does not depend on  $t$  and can be approximated by  $1/F_i$ . A third implicit assumption is that the quality inspector spends the same amount of time inspecting each failure mode. A direction for further research would be to replace these and other simplifying assumptions by more realistic assumptions and come up with more complicated but also more realistic models.



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